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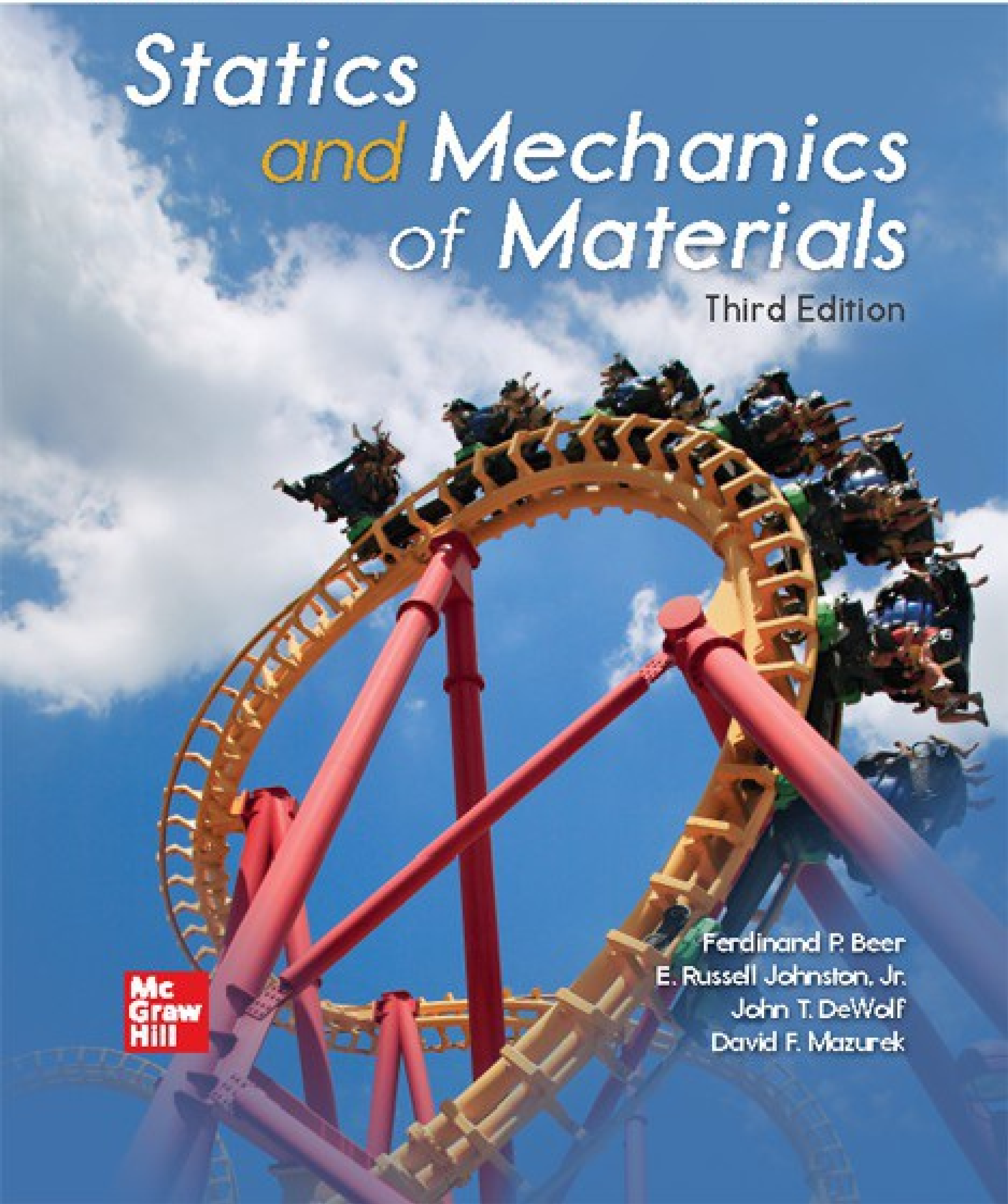
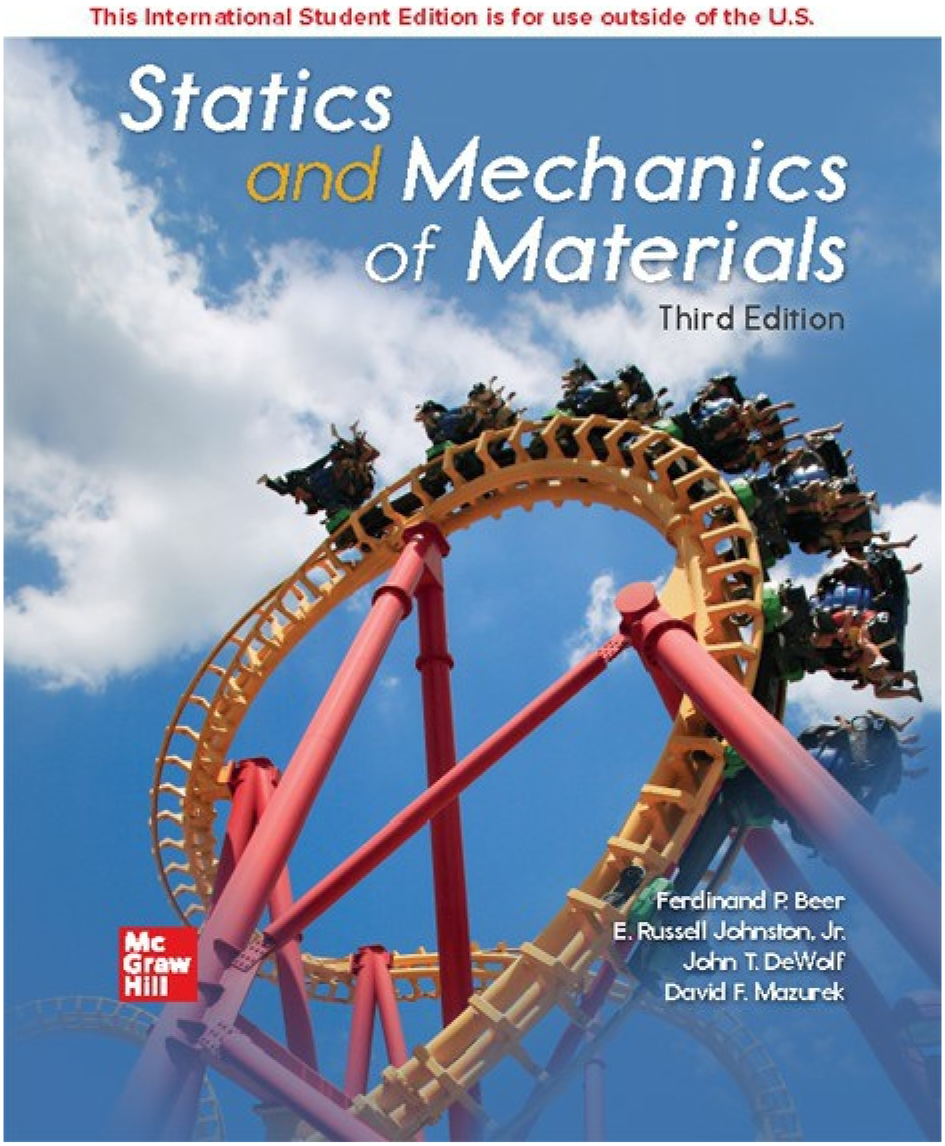


Statics and Mechanics of Materials

Third Edition

Ferdinand P. Beer
E. Russell Johnston, Jr.
John T. DeWolf
David F. Mazurek

Mc Graw Hill



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Statics and Mechanics of Materials

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STATICS AND MECHANICS OF MATERIALS

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John T. DeWolf, Emeritus Professor of Civil Engineering at the University of Connecticut, joined the Beer and Johnston team as an author on the second edition of *Mechanics of Materials*. John holds a B.S. degree in civil engineering from the University of Hawaii and M.E. and Ph.D. degrees in structural engineering from Cornell University. He is a Fellow of the American Society of Civil Engineers and a member of the Connecticut Academy of Science and Engineering. He is a registered Professional Engineer and a member of the Connecticut Board of Professional Engineers. He was selected as a University of Connecticut Teaching Fellow in 2006. Professional interests include elastic stability, bridge monitoring, and structural analysis and design.

David F. Mazurek, Professor of Civil Engineering at the United States Coast Guard Academy, joined the Beer and Johnston team on the eighth edition of *Statics* and the fifth edition of *Mechanics of Materials*. David holds a B.S. degree in ocean engineering and an M.S. degree in civil engineering from the Florida Institute of Technology and a Ph.D. degree in civil engineering from the University of Connecticut. He is a Fellow of the American Society of Civil Engineers and a member of the Connecticut Academy of Science and Engineering. He is a registered Professional Engineer and has served on the American Railway Engineering & Maintenance-of-Way Association’s Committee 15—Steel Structures since 1991. Among his numerous awards, he was recognized by the National Society of Professional Engineers as the Coast Guard Engineer of the Year for 2015. Professional interests include bridge engineering, structural forensics, and blast-resistant design.

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Preface

Objectives

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for a course that combines statics and mechanics of materials—or strength of materials—offered to engineering students in the sophomore year.

General Approach

In this text, the study of statics and mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to clearly indicate the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this text is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies.* This approach makes it possible to consider simple, practical applications at an early stage and to postpone the introduction of the more difficult concepts. As an example, statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors are introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces, leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems.

New Concepts Are Introduced in Simple Terms. Because this text is designed for the first course in mechanics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered and by stressing methods of general applicability, a definite maturity of approach is achieved. For example, the concepts of partial constraints and statical indeterminacy are introduced early and are used throughout.

Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive,* simple applications are considered first.

As an example, the statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6. In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.

The first four chapters treating mechanics of materials (Chaps. 8, 9, 10, and 11) are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. The remaining five chapters (12 through 16) expand on what is learned in Chaps. 8 through 11. Chapter 12 begins with a discussion of the shear and bending-moment diagrams and then addresses the design of beams based on the allowable normal stress in the material used. The determination of the shearing stress in beams and thin-walled members under transverse loadings is covered in Chap. 13. Chapter 14 is devoted to the transformation of stresses and design of thin-walled pressure vessels. The determination of deflections in beams is presented in Chap. 15. Chapter 16, which treats columns, contains material on the design of steel, aluminum, and wood columns.

Each analysis is based on a few basic concepts, namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study of each type of loading is complemented by a large number of examples, sample problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus; all the elements of vector algebra necessary to the understanding of mechanics are carefully presented in Chaps. 2 and 3. In general, a greater emphasis is placed on the correct understanding of the basic mathematical concepts involved than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of the moment of an area firmly before introducing the use of integration.

Free-Body Diagrams Are Used Extensively. Throughout the text, free-body diagrams are used to determine external or internal forces. The use of "picture equations" will also help the students understand the superposition of loadings and the resulting stresses and deformations.

Design Concepts Are Discussed Throughout the Text Whenever Appropriate. A discussion of the application of the factor of safety to design can be found in Chap. 8, where the concept of allowable stress design is presented.

The SMART Problem-Solving Methodology Is Employed. Students are presented with the SMART approach for solving engineering problems, whose acronym reflects the solution steps of ***S***trategy, ***M***odeling, ***A***nalysis, and ***R***eflect and ***T***hink. This methodology is used in all Sample Problems, and it is intended that students will apply this in the solution of all assigned problems.

Case Studies. The principles developed in this text are used extensively in engineering applications, particularly for design as well as for the analysis of failures. Much can be learned from the historical successes and failures of past design, and unique insight can be gained by studying how engineers developed different products and structures. To this end, real-world Case Studies have been introduced in the text to provide relevancy and application to the principles of engineering mechanics being discussed. These are developed using the SMART problem-solving methodology to present the story behind each Case Study, as well as to analyze some aspects of the situation.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the examples, sample problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Because a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find most desirable for their class.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Because the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems (such as would be encountered in a follow-on course in dynamics), an additional calculation will be required to determine its mass in slugs (or $\mathrm{lb \cdot s^2/ft}$). The authors, therefore, believe that problem assignments should include both systems of units.

Chapter Organization and Pedagogical Features

Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems.

Chapter Lessons. The body of the text has been divided into units, each consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally can be covered in one lesson.

Concept Applications and Sample Problems. Many theory sections include Concept Applications designed to illustrate the material being presented and facilitate its understanding. The Sample Problems provided after all lessons are intended to show some of the applications of the theory to the solution of engineering problems. Because they have been set up in much the same form as students will use in solving the assigned problems, the Sample Problems serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work students should cultivate in their own solutions.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and help the students understand the basic principles used in engineering mechanics. The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty. Answers to problems are given at the end of the book, except for those with a number set in *red italics*.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in the chapter. Notes in the margin have been included to help the students organize their review work, and cross references are provided to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

New to the Third Edition

We’ve made some significant changes from the second edition of this text. The updates include:

- **Case Studies.** Case Studies have been added to all chapters to provide the student with real-world engineering problems. These address how engineers approached the evaluation of problems that occurred and how they developed new designs.
- **Text Revisions.** The authors have continued to edit the language to make the book easier to read and more student-friendly.
- **Photographs.** We have updated many of the photos appearing in the third edition.
- **Revised or New Problems.** Over 20% of the problems are revised or new to this edition.

Acknowledgments

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We also gratefully acknowledge the help, comments, and suggestions offered by the many users of previous editions of books in the Beer & Johnston Engineering Mechanics series.

John T. DeWolf
David F. Mazurek

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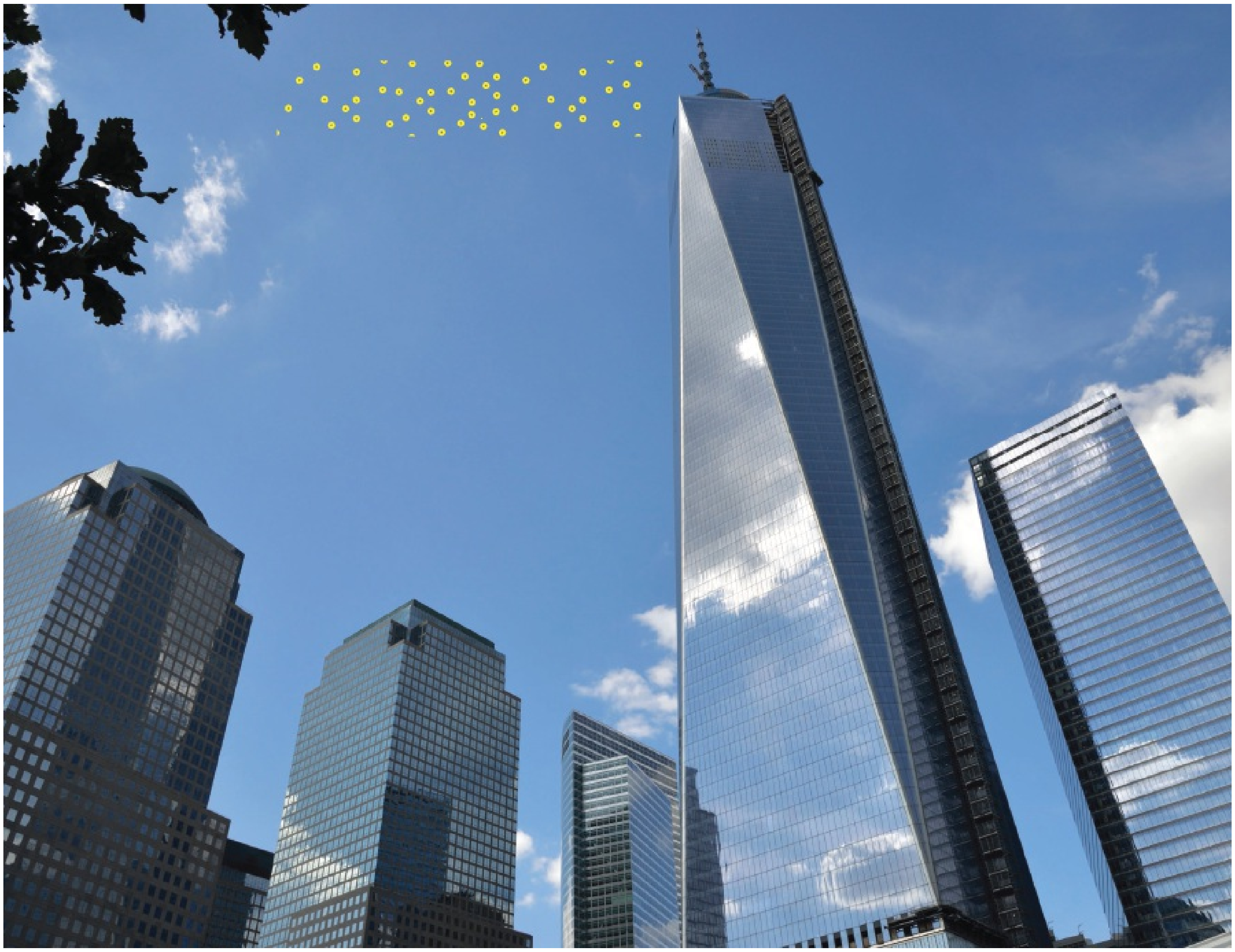
List of Symbols

a	Constant; radius; distance
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Forces; reactions at supports and connections
$A, B, C, \ldots$	Points
A, a	Area
b	Width; distance
c	Constant; distance; radius
C	Centroid
$C_1, C_2, \ldots$	Constants of integration
C_P	Column stability factor
d	Distance; diameter; depth
e	Distance; eccentricity
E	Modulus of elasticity
$\mathbf{F}$	Force; friction force
$F.S.$	Factor of safety
g	Acceleration of gravity
G	Modulus of rigidity; shear modulus
h	Distance; height
H, J, K	Points
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors along coordinate axes
$I, I_x, \ldots$	Moments of inertia
$\bar{I}$	Centroidal moment of inertia
J	Polar moment of inertia
k	Spring constant
K	Stress concentration factor; torsional spring constant
l	Length
L	Length; span
L_e	Effective length
m	Mass
$\mathbf{M}$	Couple
$M, M_x, \ldots$	Bending moment
n	Number; ratio of moduli of elasticity; normal direction
$\mathbf{N}$	Normal component of reaction
O	Origin of coordinates
p	Pressure
$\mathbf{P}$	Force; vector
P_D	Dead load (LRFD)

P_L	Live load (LRFD)
P_U	Ultimate load (LRFD)
q	Shearing force per unit length; shear flow
$\mathbf{Q}$	Force; vector
Q	First moment of area
$\bar{r}$	Centroidal radius of gyration
$\mathbf{r}$	Position vector
r_x, r_y, r_O	Radii of gyration
r	Radius; distance; polar coordinate
$\mathbf{R}$	Resultant force; resultant vector; reaction
R	Radius of earth
s	Length
$\mathbf{S}$	Force; vector
S	Elastic section modulus
t	Thickness
$\mathbf{T}$	Force; torque
T	Tension; temperature
u, v	Rectangular coordinates
$\mathbf{V}$	Vector product; shearing force
V	Volume; shear
w	Width; distance; load per unit length
$\mathbf{W}$, W	Weight; load
x, y, z	Rectangular coordinates; distances; displacements; deflections
$\bar{x}$, $\bar{y}$, $\bar{z}$	Coordinates of centroid
α, β, γ	Angles
α	Coefficient of thermal expansion; influence coefficient
γ	Shearing strain; specific weight
γ_D	Load factor, dead load (LRFD)
γ_L	Load factor, live load (LRFD)
δ	Deformation; displacement; elongation
ε	Normal strain
θ	Angle; slope
λ	Unit vector along a line
μ	Coefficient of friction
ν	Poisson's ratio
ρ	Radius of cuvature; distance; density
σ	Normal stress
τ	Shearing stress
ϕ	Angle; angle of twist; resistance factor



Renato Bordoni/Alamy Stock Photo

1
Introduction

The tallest skyscraper in the Western Hemisphere, One World Trade Center is a prominent feature of the New York City skyline. From its foundation to its structural components and mechanical systems, the design and operation of the tower is based on the fundamentals of engineering mechanics.

Objectives

- **Define** the science of mechanics and examine its fundamental principles.

- **Discuss** and compare the International System of Units and U.S. Customary Units.
- **Discuss** how to approach the solution of mechanics problems, and introduce the SMART problem-solving methodology.
- **Examine** factors that govern numerical accuracy in the solution of a mechanics problem.

Introduction

1.1 WHAT IS MECHANICS?

Mechanics is defined as the science that describes and predicts the conditions of rest or motion of bodies under the action of forces. It consists of the mechanics of *rigid bodies,* mechanics of *deformable bodies,* and mechanics of *fluids*.

The mechanics of rigid bodies is subdivided into **statics** and **dynamics**. Statics deals with bodies at rest; dynamics deals with bodies in motion. In this text, we assume bodies are perfectly rigid. In fact, actual structures and machines are never absolutely rigid; they deform under the loads to which they are subjected. However, because these deformations are usually small, they do not appreciably affect the conditions of equilibrium or the motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned. Deformations are studied in a course in mechanics of materials, which is part of the mechanics of deformable bodies. The third division of mechanics, the mechanics of fluids, is subdivided into the study of *incompressible fluids* and of *compressible fluids*. An important subdivision of the study of incompressible fluids is *hydraulics*, which deals with applications involving water.

Mechanics is a physical science, because it deals with the study of physical phenomena. However, some teachers associate mechanics with mathematics, whereas many others consider it as an engineering subject. Both of these views are justified in part. Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study. However, it does not have the *empiricism* found in some engineering sciences, i.e., it does not rely on experience or observation alone. The rigor of mechanics and the emphasis it places on deductive reasoning makes it resemble mathematics. However, mechanics is not an *abstract* or even a *pure* science; it is an *applied* science.

The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications. You need to know statics to determine how much force will be exerted on a point in a bridge design and whether the structure can withstand that force. Determining the force a dam needs to withstand from the water in a river requires statics. You need statics to calculate how much weight a crane can lift, how much force a locomotive needs to pull a freight train, or how much force a circuit board in a computer can withstand. The concepts of dynamics enable you to analyze the flight characteristics of a jet, design a building to resist earthquakes, and mitigate shock and vibration to passengers inside a vehicle. The concepts of dynamics enable you to calculate how much force you need to send a satellite into orbit, accelerate a 200,000-ton cruise ship, or design a toy truck that doesn't break. You will not learn how to do these things in this course, but the ideas and methods you learn here will be the underlying basis for the engineering applications you will learn in your work.

1.2 FUNDAMENTAL CONCEPTS AND PRINCIPLES

1.2A Mechanics of Rigid Bodies

Although the study of mechanics goes back to the time of Aristotle (384–322 b.c.) and Archimedes (287–212 b.c.), not until Newton (1642–1727) did anyone develop a satisfactory formulation of its fundamental principles. These principles were later modified by d'Alembert, Lagrange, and Hamilton. Their validity remained unchallenged until Einstein formulated his **theory of relativity** (1905). Although its limitations have now been recognized, **newtonian mechanics** still remains the basis of today's engineering sciences.

The basic concepts used in mechanics are *space*, *time*, *mass*, and *force*. These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study of mechanics.

The concept of **space** is associated with the position of a point *P*. We can define the position of *P* by providing three lengths measured from a certain reference point, or *origin,* in three given directions. These lengths are known as the *coordinates* of *P*.

To define an event, it is not sufficient to indicate its position in space. We also need to specify the **time** of the event.

We use the concept of **mass** to characterize and compare bodies on the basis of certain fundamental mechanical experiments. Two bodies of the same mass, for example, are attracted by the earth in the same manner; they also offer the same resistance to a change in translational motion.

A **force** represents the action of one body on another. A force can be exerted by actual contact, like a push or a pull, or at a distance, as in the case of gravitational or magnetic forces. A force is characterized by its *point of application,* its *magnitude,* and its *direction;* a force is represented by a *vector* (Sec. 2.1B).

In newtonian mechanics, space, time, and mass are absolute concepts that are independent of each other. (This is not true in **relativistic mechanics**, where the duration of an event depends upon its position and the mass of a body varies with its velocity.) On the other hand, the concept of force is not independent of the other three. Indeed, one of the fundamental principles of newtonian mechanics listed below is that the resultant force acting on a body is related to the mass of the body and to the manner in which its velocity varies with time.

In this text, you will study the conditions of rest or motion of particles and rigid bodies in terms of the four basic concepts we have introduced. By **particle**, we mean a very small amount of matter, which we assume occupies a single point in space. A **rigid body** consists of a large number of particles occupying fixed positions with respect to one another. The study of the mechanics of particles is clearly a prerequisite to that of rigid bodies. Besides, we can use the results obtained for a particle directly in a large number of problems dealing with the conditions of rest or motion of actual bodies.

The study of elementary mechanics rests on six fundamental principles, based on experimental evidence.

- **The Parallelogram Law for the Addition of Forces.** Two forces acting on a particle may be replaced by a single force, called their *resultant,* obtained by drawing the diagonal of the parallelogram with sides equal to the given forces (Sec. 2.1A).
- **The Principle of Transmissibility.** The conditions of equilibrium or of motion of a rigid body remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action (Sec. 3.1B).
- **Newton's Three Laws of Motion.** Formulated by Sir Isaac Newton in the late 17th century, these laws can be stated as follows:

  FIRST LAW. If the resultant force acting on a particle is zero, the particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion) (Sec. 2.3B).

  SECOND LAW. If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

  This law can be stated as

  $$\mathbf{F} = m\mathbf{a} \tag{1.1}$$

  where **F**, *m*, and **a** represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle expressed in a consistent system of units.

  THIRD LAW. The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense (Chap. 6, Introduction).
- **Newton's Law of Gravitation.** Two particles of mass *M* and *m* are mutually attracted with equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ of magnitude *F* (Fig. 1.1), given by the formula

$$F = G\frac{Mm}{r^2} \tag{1.2}$$

where r = the distance between the two particles and G = a universal constant called the *constant of gravitation.* Newton's law of gravitation introduces the idea of an action exerted at a distance and extends the range of application of Newton's third law: the action **F** and the reaction $-\mathbf{F}$ in Fig. 1.1 are equal and opposite, and they have the same line of action.

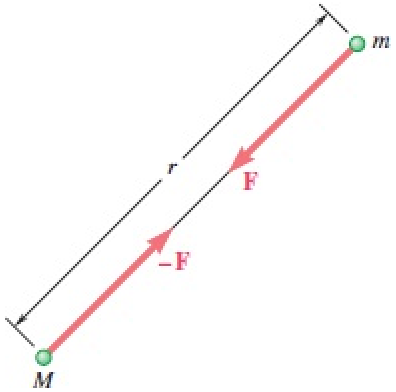


Fig. 1.1 From Newton's law of gravitation, two particles of masses *M* and *m* exert forces upon each other of equal magnitude, opposite direction, and the same line of action. This also illustrates Newton's third law of motion.

A particular case of great importance is that of the attraction of the earth on a particle located on its surface. The force **F** exerted by the earth on the particle is defined as the **weight W** of the particle. Suppose we set *M* equal to the mass of the earth, *m* equal to the mass of the particle, and *r* equal to the earth's radius *R*. Then, introducing the constant

$$g = \frac{GM}{R^2} \tag{1.3}$$

we can express the magnitude *W* of the weight of a particle of mass *m* as†

$$W = mg \tag{1.4}$$

The value of *R* in Eq. (1.3) depends upon the elevation of the point considered; it also depends upon its latitude, because the earth is not truly spherical. The value of *g* therefore varies with the position of the point considered. However, as long as the point actually remains on the earth's surface, it is sufficiently accurate in most engineering computations to assume that g equals $9.81\ \mathrm{m/s^2}$ or $32.2\ \mathrm{ft/s^2}$.

The principles we have just listed will be introduced in the course of our study of mechanics as they are needed. The statics of particles carried out in Chap. 2 will be based on the parallelogram law of addition and on Newton's first law alone. We introduce the principle of transmissibility in Chap. 3 as we begin the study of the statics of rigid bodies, and we bring in Newton's third law in Chap. 6 as we analyze the forces exerted on each other by the various members forming a structure.

As noted earlier, the six fundamental principles listed previously are based on experimental evidence. Except for Newton's first law and the principle of transmissibility, they are independent principles that cannot be derived mathematically from each other or from any other elementary physical principle. On these principles rests most of the intricate structure of newtonian mechanics. For more than two centuries, engineers have solved a tremendous number of problems dealing with the conditions of rest and motion of rigid bodies, deformable bodies, and fluids by applying these fundamental principles. Many of the solutions obtained could be checked experimentally, thus providing a further verification of the principles from which they were derived. Only in the 20th century has Newton's mechanics found to be at fault, in the study of the motion of atoms and the motion of the planets, where it must be supplemented by the theory of relativity. On the human or engineering scale, however, where velocities are small compared with the speed of light, Newton's mechanics have yet to be disproved.

Photo 1.1 When in orbit of the earth, people and objects are said to be *weightless*, even though the gravitational force acting is approximately 90% of that experienced on the surface of the earth. This apparent contradiction can be resolved in a course on dynamics when Newton's second law is applied to the motion of particles.

Source: NASA

1.2B Mechanics of Deformable Bodies

The concepts needed for mechanics of deformable bodies, also referred to as *mechanics of materials,* are necessary for analyzing and designing various machines and load-bearing structures. These concepts involve the determination of *stresses* and *deformations.*

In Chaps. 8 through 16, the analysis of stresses and the corresponding deformations will be developed for structural members subject to axial loading, torsion, and bending. This requires the use of basic concepts involving the conditions of equilibrium of forces exerted on the member, the relations existing between stress and deformation in the material, and the conditions imposed by the supports and loading of the member. Later chapters expand on these subjects, providing a basis for designing both structures that are statically determinant and those that are indeterminant, i.e., structures in which the internal forces cannot be determined from statics alone.

1.3 SYSTEMS OF UNITS

Associated with the four fundamental concepts just discussed are the so-called *kinetic units*, i.e., the units of *length, time, mass*, and *force*. These units cannot be chosen independently if Eq. (1.1) is to be satisfied. Three of the units may be defined arbitrarily; we refer to them as **basic units**. The fourth unit, however, must be chosen in accordance with Eq. (1.1) and is referred to as a **derived unit**. Kinetic units selected in this way are said to form a **consistent system of units**.

International System of Units (SI Units).[†] In this system, which will be in universal use after the United States has completed its conversion to SI units, the base units are the units of length, mass, and time, and they are called, respectively, the **meter** (m), the **kilogram** (kg), and the **second** (s). All three are arbitrarily defined. The second was originally chosen to represent 1/86 400 of the mean solar day, but it is now defined as the duration of 9 192 631 770 cycles of the radiation corresponding to the transition between two levels of the fundamental state of the cesium-133 atom. The meter, originally defined as one ten-millionth of the distance from the equator to either pole, is now defined as 1 650 763.73 wavelengths of the orange-red light corresponding to a certain transition in an atom of krypton-86. (The newer definitions are much more precise and with today's modern instrumentation, are easier to verify as a standard.) The kilogram, which is approximately equal to the mass of 0.001 m^3 of water, is defined as the mass of a platinum-iridium standard kept at the International Bureau of Weights and Measures at Sèvres, near Paris, France. The unit of force is a derived unit. It is called the **newton** (N) and is defined as the force that gives an acceleration of 1 $\mathrm{m/s^2}$ to a body of mass 1 kg (Fig. 1.2). From Eq. (1.1), we have

$$1\ \mathrm{N} = (1\ \mathrm{kg})(1\ \mathrm{m/s^2}) = 1\ \mathrm{kg\cdot m/s^2} \tag{1.5}$$

The SI units are said to form an *absolute* system of units. This means that the three base units chosen are independent of the location where measurements are made. The meter, the kilogram, and the second may be used anywhere on the earth; they may even be used on another planet and still have the same significance.

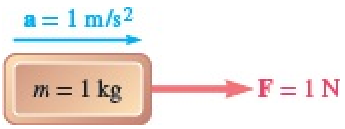


Fig. 1.2 A force of 1 newton applied to a body of mass 1 kg provides an acceleration of $1\ \mathrm{m/s^2}$.

The *weight* of a body, or the *force of gravity* exerted on that body, like any other force, should be expressed in newtons. From Eq. (1.4), it follows that the weight of a body of mass 1 kg (Fig. 1.3) is

$$\begin{aligned} W &= mg \\ &= (1\ \mathrm{kg})\left(9.81\ \mathrm{m/s^2}\right) \\ &= 9.81\ \mathrm{N} \end{aligned}$$

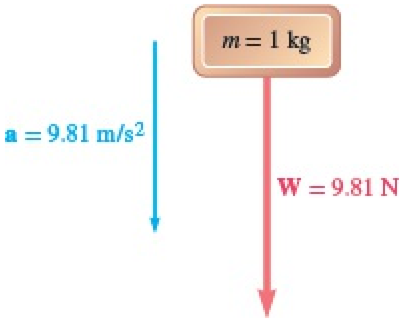
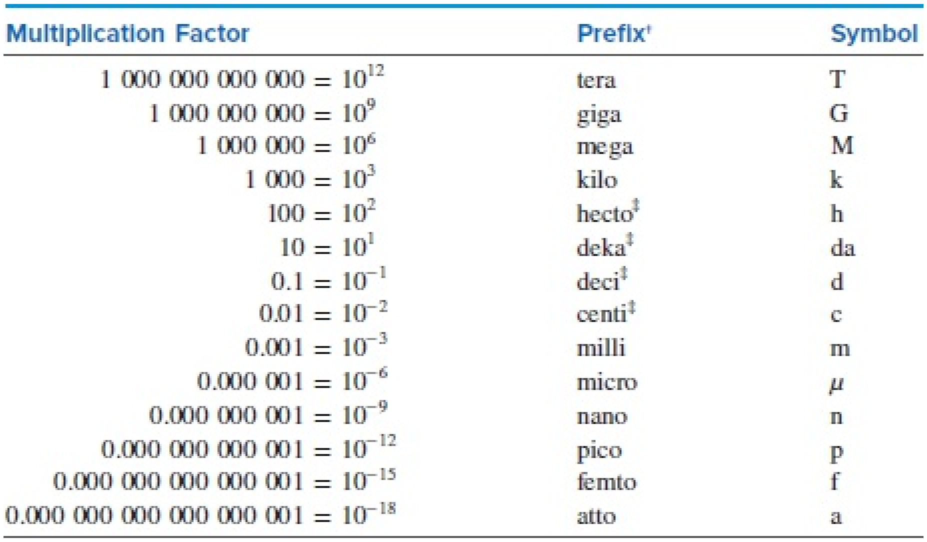


Fig. 1.3 A body of mass 1 kg experiencing an acceleration due to gravity of $9.81\ \mathrm{m/s^2}$ has a weight of 9.81 N.

Multiples and submultiples of the fundamental SI units are denoted through the use of the prefixes defined in Table 1.1. The multiples and submultiples of the units of length, mass, and force most frequently used in engineering are, respectively, the *kilometer* (km) and the *millimeter* (mm); the *megagram*‡ (Mg) and the *gram* (g); and the *kilonewton* (kN). According to Table 1.1, we have

$$\begin{aligned} 1\ \mathrm{km} &= 1000\ \mathrm{m} & 1\ \mathrm{mm} &= 0.001\ \mathrm{m} \\ 1\ \mathrm{Mg} &= 1000\ \mathrm{kg} & 1\ \mathrm{g} &= 0.001\ \mathrm{kg} \\ 1\ \mathrm{kN} &= 1000\ \mathrm{N} \end{aligned}$$

Table 1.1 Sl Prefixes



Multiplication Factor	Prefix†	Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto‡	h
10 = 10^{1}	deka‡	da
0.1 = 10^{-1}	deci‡	d
0.01 = 10^{-2}	centi‡	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

†The first syllable of every prefix is accented, so that the prefix retains its identity. Thus, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.

‡The use of these prefixes should be avoided, except for the measurement of areas and volumes and for the nontechnical use of centimeter, as for body and clothing measurements.

The conversion of these units into meters, kilograms, and newtons, respectively, can be effected by simply moving the decimal point three places to the right or to the left. For example, to convert 3.82 km into meters, move the decimal point three places to the right:

$$3.82\text{ km} = 3820\text{ m}$$

Similarly, to convert 47.2 mm into meters, move the decimal point three places to the left:

$$47.2\,\text{mm} = 0.0472\text{ m}$$

Using engineering notation, you can also write

$$\begin{aligned} 3.82\,\text{km} &= 3.82 \times 10^{3}\text{ m} \\ 47.2\,\text{mm} &= 47.2 \times 10^{-3}\text{ m} \end{aligned}$$

The multiples of the unit of time are the *minute* (min) and the *hour* (h). Because 1 min = 60 s and

1 h = 60 min = 3600 s, these multiples cannot be converted as readily as the others.

By using the appropriate multiple or submultiple of a given unit, you can avoid writing very large or very small numbers. For example, it is usually simpler to write 427.2 km rather than 427 200 m and 2.16 mm rather than 0.002 16 m.†

Units of Area and Volume. The unit of area is the *square meter* (m^2), which represents the area of a square of side 1 m; the unit of volume is the *cubic meter* $\left(m^3\right)$, which is equal to the volume of a cube of side 1 m. To avoid exceedingly small or large numerical values when computing areas and volumes, we use systems of subunits obtained by respectively squaring and cubing not only the millimeter, but also two intermediate submultiples of the meter: the *decimeter* (dm) and the *centimeter* (cm). By definition,

$$\begin{aligned} 1\ \mathrm{dm} &= 0.1\ \mathrm{m} = 10 - 1\ \mathrm{m} \\ 1\ \mathrm{cm} &= 0.01\ \mathrm{m} = 10^{-2}\ \mathrm{m} \\ 1\ \mathrm{mm} &= 0.001\ \mathrm{m} = 10^{-3}\ \mathrm{m} \end{aligned}$$

Therefore, the submultiples of the unit of area are

$$\begin{aligned} 1\ \mathrm{dm}^2 &= (1\ \mathrm{dm})^2 = \left(10^{-1}\mathrm{m}\right)^2 = 10^{-2}\ \mathrm{m}^2 \\ 1\ \mathrm{cm}^2 &= (1\ \mathrm{cm})^2 = \left(10^{-2}\mathrm{m}\right)^2 = 10^{-4}\ \mathrm{m}^2 \\ 1\ \mathrm{mm}^2 &= (1\ \mathrm{mm})^2 = \left(10^{-3}\mathrm{m}\right)^2 = 10^{-6}\ \mathrm{m}^2 \end{aligned}$$

Similarly, the submultiples of the unit of volume are

$$\begin{aligned} 1\ \mathrm{dm}^3 &= (1\ \mathrm{dm})^3 = \left(10^{-1}\mathrm{m}\right)^3 = 10^{-3}\ \mathrm{m}^3 \\ 1\ \mathrm{cm}^3 &= (1\ \mathrm{cm})^3 = \left(10^{-2}\mathrm{m}\right)^3 = 10^{-6}\ \mathrm{m}^3 \\ 1\ \mathrm{mm}^3 &= (1\ \mathrm{mm})^3 = \left(10^{-3}\mathrm{m}\right)^3 = 10^{-9}\ \mathrm{m}^3 \end{aligned}$$

Note that when measuring the volume of a liquid, the cubic decimeter $\left(\mathrm{dm}^3\right)$ is usually referred to as a *liter* (L).

Table 1.2 shows other derived SI units used to measure the moment of a force, the work of a force, etc. Although we will introduce these units in later chapters as they are needed, we should note an important rule at this time: When a derived unit is obtained by dividing a base unit by another base unit, you may use a prefix in the numerator of the derived unit, but not in its denominator. For example, the constant *k* of a spring that stretches 20 mm under a load of 100 N is expressed as